

Sydney Technical High School



2011
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (120)

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 (12 Marks)	Use a Separate Sheet of paper	Marks
(a) Factorise $3x^2 - 5x - 2$		2
(b) Simplify $2x^2y - yx^2 + xy^2 + 2y^2x$		1
(c) Express $\frac{3\pi}{8}$ radians in degrees and minutes.		1
(d) If $\frac{4}{2-\sqrt{3}} = a + b\sqrt{3}$ find the values of a and b . Ans		2
(e) Solve $ 2x + 5 < 3$		2
(f) Find the period and amplitude for the graph of $3y = \sin\left(2x - \frac{\pi}{4}\right)$.		2
(g) Solve $9^{2x-3} = 27^x$		2

Question 2 (12 Marks)

Use a Separate Sheet of paper

Marks(a) Differentiate with respect to x :

i) $(3x^2 + 7)^6$

2

ii) $\frac{\sin 2x}{e^{2x}}$

2

(b) i) Evaluate $\int \frac{dx}{3x+5}$

2

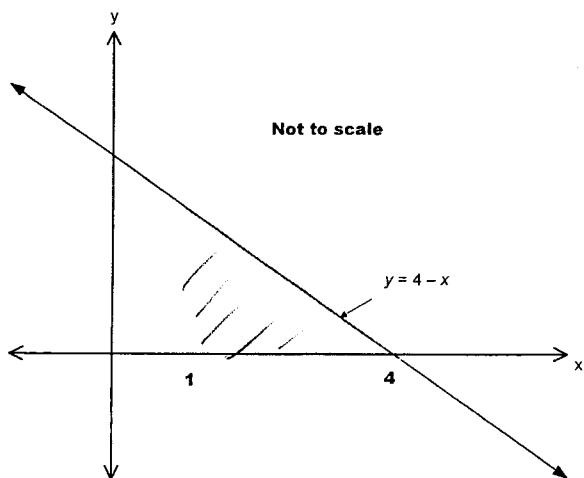
ii) $\int_0^1 e^{3x} dx$

2

(c) $y = 4 - x$ is shown on the graph.

3

Calculate the volume of the solid formed when the area bounded by the function, x axis and $x = 1$ is rotated around the x axis.

(d) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$

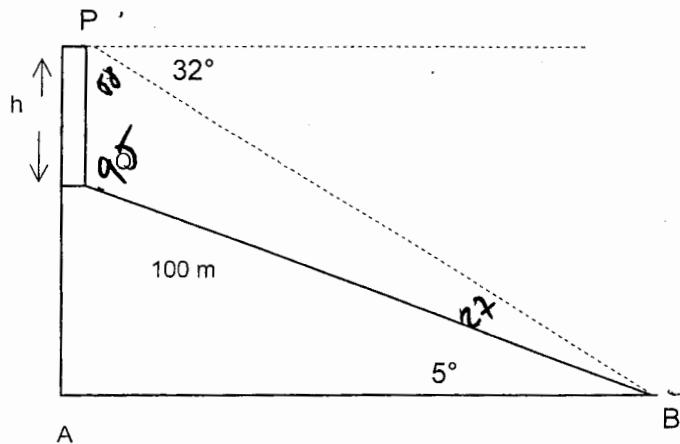
1

Question 3 (12 Marks)

Use a Separate Sheet of paper

Marks

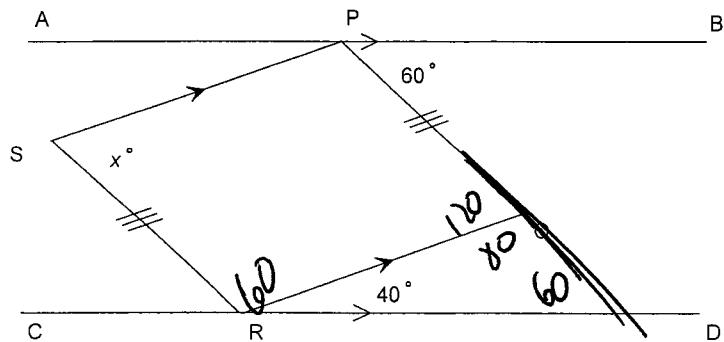
- (a) A vertical tower PQ is at the top of a hill BQ, where Q is 100 metres from the base B. The hill is inclined at 5° to the horizontal as shown below. From the top of the tower the angle of depression to the point B is 32° .



- (i) Copy the diagram and mark on it all of the relevant information. 1
- (ii) Calculate the height (h) of the tower, to the nearest metre. 2
- (b) For the arithmetic sequence 2, 7, 12, 17,
- (i) Find a value of the n th Term 1
- (ii) Find the 23rd term 1
- (iii) Find the sum of the first 47 terms 1

- (c) In the diagram, $AB \parallel CD$ and PQRS is a parallelogram.

3



Find the value of x , giving reasons.

()

- d) Find the equation of the tangent to the curve $y = \sin 3x$ at the point where $x = \frac{\pi}{3}$

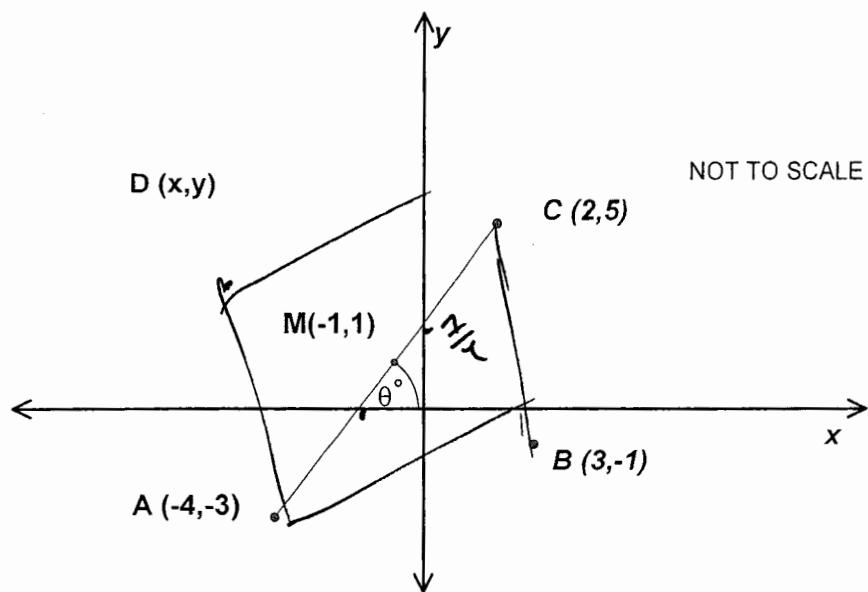
3

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Question 4 (12 Marks)

Use a Separate Sheet of paper

Marks



You are given that $M(-1, 1)$ is the midpoint of AC

- a) Find the coordinates of D such that M is the midpoint of BD 2
- b) Using the facts already known, explain why $ABCD$ is a parallelogram. 1
- c) Find the size of θ to the nearest degree. 2
- d) Show that the equation of AC is $4x - 3y + 7 = 0$ 2
- e) Find the perpendicular distance between B and the line AC 3
- f) **Copy the diagram into your answer booklet.**
Shade the region inside the quadrilateral $ABCD$, which satisfies the inequality $4x - 3y + 7 \leq 0$ 2

Question 5 (12 Marks)	Use a Separate Sheet of paper	Marks
(a) (i)	Find the value(s) of k for which $x^2 + (2 - k)x + 2.25 = 0$ has equal roots	2
(ii)	Find the value(s) of k for which $y = kx + 1$ is a tangent to $y = x^2 + 2x + 3.25$	1
(b)	Consider the curve $y = x^4 - \frac{4}{3}x^3 - 2x^2 + 4x + 3$	
(i)	Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for this function	2
(ii)	Show that $x = -1$ and $x = 1$ satisfy $\frac{dy}{dx} = 0$ and find the y coordinates.	2
(iii)	Find the x coordinates of the two points of inflexion.	1
(iv)	Determine the nature of each of the stationary points.	2
(v)	Sketch the curve for the domain $-2 \leq x \leq 2$	2

Question 6 (12 Marks)

Use a Separate Sheet of paper

Marks

- (a) The equation of a parabola is given by
- $x^2 - 4x - 2y + 8 = 0$
- . Find the:

(i) Vertex

2

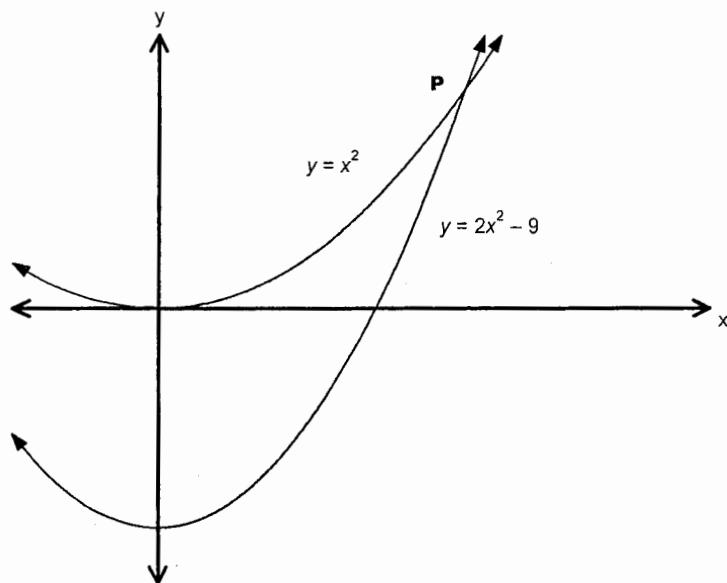
(ii) Focus

2

(iii) Equation of the normal to the parabola at the point (0, 4).

2

- (b) P is the point of intersection of
- $y = x^2$
- and
- $y = 2x^2 - 9$



(i) Find the coordinates of P.

2

(ii) Find the area of the shaded region.

2

- (c) Evaluate
- $\sum_{k=4}^{20} 2k - 5$

2

Question 7 (12 Marks)

Use a Separate Sheet of paper

Marks

(a) Show that $\frac{(1 + \tan^2 \theta)\cot \theta}{\cosec^2 \theta} = \tan \theta$ X 3

(b) Solve $2\log_a x - \log_a 4 = 2\log_a 8$ X 3

(c) If α and β are the roots of the equation $3x^2 - 2x - 4 = 0$, find:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $(4 - \alpha)(4 - \beta)$ 2

(d) Use the table to find an approximation to the value of the definite integral

$$\int_3^{4.5} f(x)dx,$$

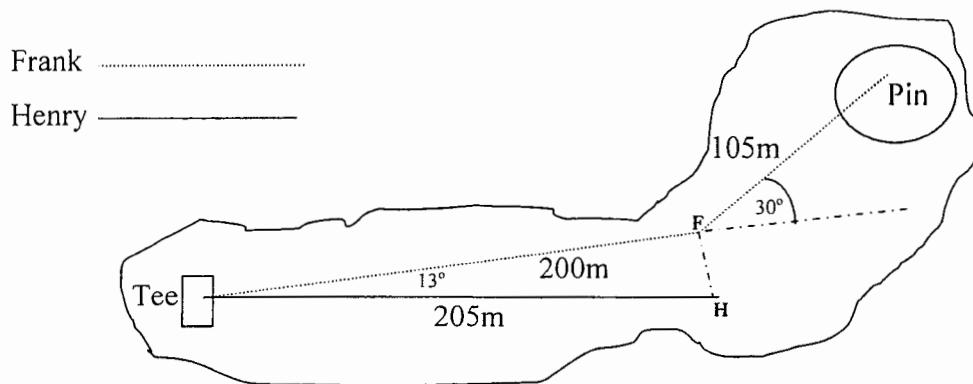
using Simpson's Rule. Give your answer correct to 3 significant figures.

2

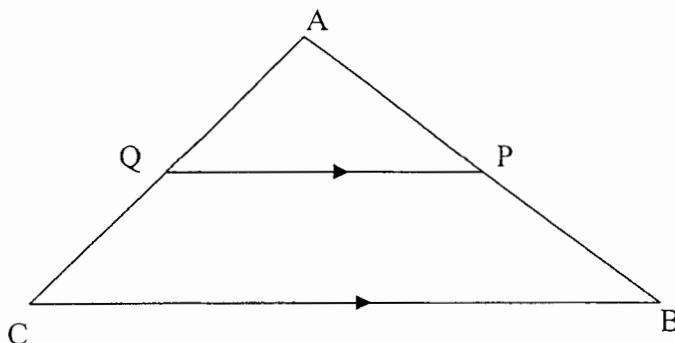
x	3	3.25	3.5	3.75	4	4.25	4.5
$f(x)$	1.0	0.8	0.65	0.55	0.5	0.48	0.45

Question 8 (12 Marks) Use a Separate Sheet of paper Marks

- (a) The 18th hole at Royal Maples is a dogleg to the left. Frank hits a 200m drive then turns left 30° and hits a 105m shot to the pin.
- (i) What is the straight line distance from the tee to the pin? 3
- (ii) Henry hits his drive a distance of 205m and to the right of Frank's drive line by 13°. Show that the triangle formed by the two initial drives is approximately right angled. ~~the~~



- (b) In the diagram below, P is the midpoint of the side AB of the $\triangle ABC$. PQ is drawn parallel to BC.



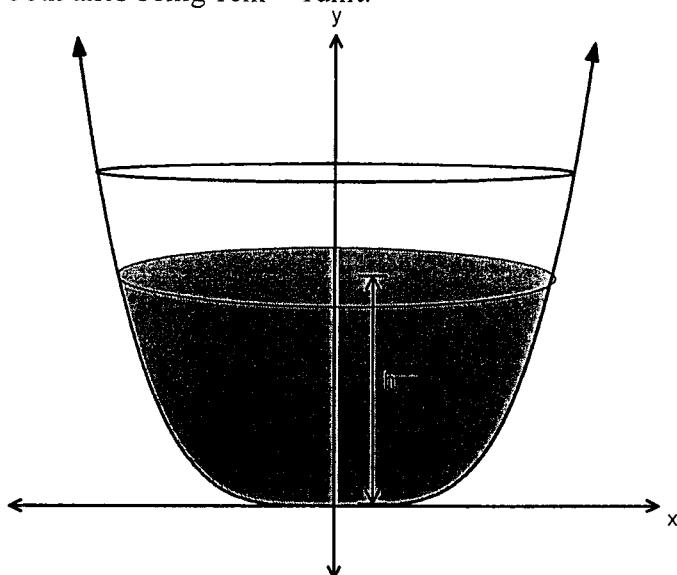
- (i) Prove that $\triangle ABC \parallel \triangle APQ$. 2
- (ii) Explain why Q is the midpoint of AC. 2
- (c) On the same diagram sketch the graphs of $y = \sin x$ and $y = 2 \sin x + 1$ ~~the~~ 3
- $0 \leq x \leq 2\pi$

Question 9 (12 Marks)

Use a Separate Sheet of paper

Marks

- (a) A wine glass is formed by rotating part of the curve $16y = x^4$ about the y axis, the scales on both axes being 1cm = 1unit.



3

- (i) If the depth of the wine is h cm, show that the volume of wine is $\frac{8\pi h^5}{3}$ mL. 2

- (ii) If the volume is 120 mL, find h correct to one decimal place 2

(b) Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x) dx$ 2

(c) Solve: $2 \sin^2 x - 3 \sin x - 2 = 0$ for $0 \leq x \leq 2\pi$ 2

3

- (d) (i) Sketch $y = \ln(x+1)$ 1

- (ii) Find the area bounded by the curve, the x-axis and the line $x = 2$ 3

Question 10 (12 Marks)

Use a Separate Sheet of paper

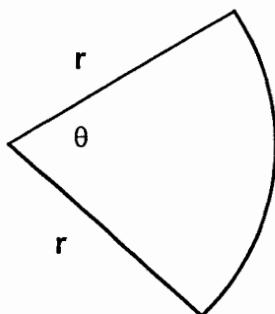
Marks

- (a) (i) Paula is in a superannuation fund to which she contributes \$250.00 at the beginning of each month for 30 years. The fund pays 6.6% pa compounded monthly. If the fund matures at the end of the last month of the 30th year, find the total value of the fund at maturity. 3

- (ii) After receiving the payout from the fund, Paula sells her Audi for \$30 000 and invests the total of the two assets in an account that earns interest at 6.6% p.a. compounded monthly. How much will the investment be worth after a further 10 years? 2

- (b) Arcsec Landscaping Company are designing a garden bed for a local park in the shape of a sector with radius r and sector angle θ . ()

They have a total of 375 metres of garden edging materials to use as the perimeter of the garden bed.

**NOT TO SCALE**

- (i) Show that the area A of the garden bed is given by $A = \frac{r}{2}(375 - 2r)$. 2 ()
- (ii) Finding the greatest area of the garden bed which can be made using 375 metres of edging material? 3
- (iii) After inspecting the location for the garden bed the designers calculate that the sector angle for the garden must be less than 110°. Can they still create the garden bed with a maximum area found in (ii)? Justify your answer. 2

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Question 1

$$i) 3x^2 - 5x - 2 = (3x+1)(x-2)$$

$$ii) 2x^2y - yx^2 + xy^2 + 2y^2x \\ = x^2y + 3xy^2$$

$$iii) \frac{3\pi}{8} = 67^\circ 30'$$

$$iv) \frac{4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{8+4\sqrt{3}}{1}$$

$$a=8 \quad b=4$$

$$v) |2x+5| < 3 \\ -3 < 2x+5 < 3 \\ -8 < 2x < -2 \\ -4 < x < -1$$

$$vi) 3y = \sin(2x - \frac{\pi}{4}) \\ y = \frac{1}{3} \sin(2x - \frac{\pi}{4})$$

$$\text{Amplitude} = \frac{1}{3} \\ \text{Period} = \frac{2\pi}{2} = \pi$$

$$7^{2x-3} = 27^x$$

$$(3^2)^{2x-3} = 3^{3x}$$

$$4x-6 = 3x$$

$$x = 6$$

Question 2

$$a)i) y = (3x^2 + 7)^6$$

$$\frac{dy}{dx} = 6(3x^2 + 7)^5 \times 6x \\ = 36x(3x^2 + 7)^5$$

$$a.ii) y = \frac{\sin 2x}{e^{2x}}$$

$$u = \sin 2x \quad v = e^{-2x} \\ du = 2\cos 2x \quad dv = -2e^{-2x}$$

$$\frac{dy}{dx} = \frac{e^{2x} \cdot 2\cos 2x - \sin 2x \cdot -2e^{2x}}{(e^{2x})^2} \\ = \frac{2e^{2x}[\cos 2x + \sin 2x]}{(e^{2x})^2} \\ = \frac{2[\cos 2x + \sin 2x]}{e^{2x}}$$

$$b.i) \int \frac{dx}{3x+5} = \frac{1}{3} \int \frac{3}{3x+5} dx \\ = \frac{1}{3} \ln(3x+5) + C$$

$$b.ii) \int_0^1 e^{3x} dx = \left[\frac{1}{3} e^{3x} \right]_0^1 \\ = \frac{1}{3} e^3 - \frac{1}{3} \\ = \frac{1}{3} (e^3 - 1)$$

$$c) V = \pi \int_1^4 y^2 dx$$

$$y = 4-x \\ y^2 = (4-x)^2$$

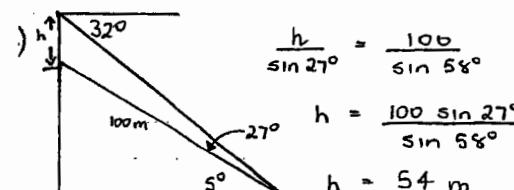
$$V = \pi \int_1^4 (4-x)^2 dx$$

$$= \pi \int \left[\frac{(4-x)^3}{3} \right]_1^4$$

$$= \pi \left[0 - (-9) \right] \\ = 9\pi \text{ units}^3$$

$$d) \lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \frac{\frac{3}{2} \times \sin 3x}{3x} \\ = \frac{\frac{3}{2} \times 1}{3} \\ = \frac{3}{2}$$

Question 3.



$$\frac{h}{\sin 27^\circ} = \frac{100}{\sin 58^\circ}$$

$$h = \frac{100 \sin 27^\circ}{\sin 58^\circ} \\ h = 54 \text{ m}$$

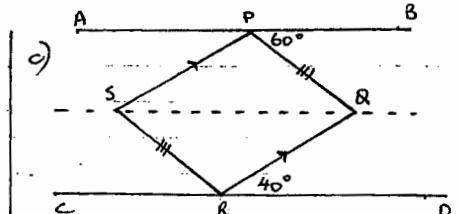
$$b).i) T_n = S_n - 3$$

$$ii) T_{23} = 5 \times 23 - 3 = 112$$

$$iii) S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{47}{2} [2 \times 2 + (47-1) \times 5]$$

$$= 5499$$



Construct a line through SR, \parallel to AB and CD

$$\angle SQR = \angle QRD = 40^\circ$$

(alternate L's are equal in parallel lines $SP \parallel QR$)

$$\angle SPQ = \angle BPQ = 60^\circ$$

(alternate L's are equal in parallel lines $SA \parallel QR$)

$$\therefore \angle PQR = 100^\circ$$

$$\therefore x = \angle PSR = 100^\circ$$

(opposite L's of a parallelogram $PQRS$ are equal)

$$d) y = \sin 3x$$

$$\frac{dy}{dx} = 3 \cos 3x$$

$$\text{When } x = \frac{\pi}{3}$$

$$\frac{dy}{dx} = 3 \cos 3 \times \frac{\pi}{3}$$

$$= -3$$

$$y = 0$$

$$y - 0 = -3(x - \frac{\pi}{3})$$

$$y = -3x + \pi$$

Question 4

Matrix

a) $\frac{x+3}{2} = -1 \quad \frac{y-1}{2} = 1$
 $x+3 = -2 \quad y-1 = 2$
 $x = -5 \quad y = 3$
 $D(-5, 3)$

2

b) Its diagonals bisect each other.

c) Gradient of MC = $\frac{5-1}{2+1} = \frac{4}{3}$
 $\tan \theta = \frac{4}{3}$
 $\theta = 53^\circ$

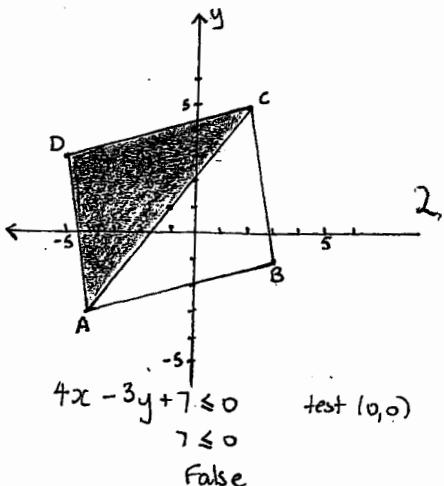
2

d) $y-5 = \frac{4}{3}(x-2)$
 $3y-15 = 4x-8$
 $4x-3y+7=0$

2

e) perpendicular distance
 $= \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$
 $= \frac{|4x_3 - 3x_1 + 7|}{\sqrt{4^2 + 3^2}}$
 $= 4 \cdot 4$

3

Question 5

ai) $x^2 - (2-k)x + 2.25 = 0$
 $\Delta = 0 \quad \Delta = (2-k)^2 - 4 \times 2.25 \times 1$
 $0 = (2-k)^2 - 9$
 $9 = (2-k)^2$
 $\pm 3 = 2-k$
 $k = -1 \quad \text{or } k = 5$

aii) If $kx+1 = x^2 + 2x + 3.25$
 $Kx = x^2 + 2x + 2.25$
 $0 \leq x^2 + (2k-2)x + 2.25$
 $0 = x^2 + (2-k)x + 2.25$

So tangent if $k=-1$ or 5

2. bi) $y = x^4 - \frac{4}{3}x^3 - 2x^2 + 4x + 3$
 $y' = 4x^3 - 4x^2 - 4x + 4$
 $y'' = 12x^2 - 8x - 4$

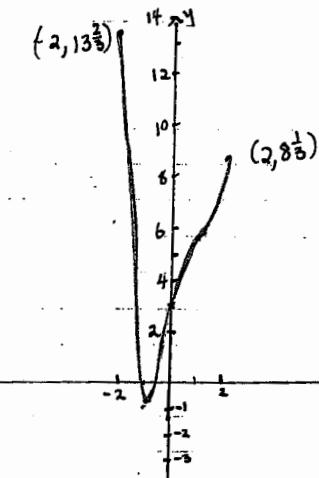
bii) $f'(1) = 4(1)^3 - 4(1)^2 - 4(1) + 4$
 $= 0$
 $f'(-1) = 4(-1)^3 - 4(-1)^2 - 4(-1) + 4$
 $= 0$

When $x=1 \quad y = 4\frac{2}{3}$
 $x=-1 \quad y = -\frac{2}{3}$

biii) $y'' = 12x^2 - 8x - 4$
Point of inflection occurs when
 $y''=0$
 $12x^2 - 8x - 4 = 0$
 $3x^2 - 2x - 1 = 0$
 $(3x+1)(x-1) = 0$
 $x = -\frac{1}{3} \quad x = 1$

iv) $(1, 4\frac{2}{3})$ is a horizontal point of inflection

When $x=-1$
 $f''(-1) > 0$
 \therefore minimum at $(-1, -\frac{2}{3})$

Question 6

a) $x^2 - 4x - 2y + 8 = 0$
 $x^2 - 4x = 2y - 8$
 $x^2 - 4x + 4 = 2y - 8 + 4$
 $(x-2)^2 = 2y - 4$
 $(x-2)^2 = 2(y-2)$

i) Vertex $(2, 2)$

ii) Focus $4a = 2$
 $a = \frac{1}{2}$

iii) $x^2 - 4x + 8 = 2y$
 $y = \frac{1}{2}(x^2 - 4x + 8)$
 $y' = x - 2$

When $x=0$
 $m_1 = -2$
 $m_2 = \frac{1}{2}$
Equation of normal
 $y - 4 = \frac{1}{2}(x - 0)$
 $y = \frac{1}{2}x + 4$

b.i) $y = x^2 \quad y = 2x^2 - 9$
 $x^2 = 2x^2 - 9$
 $9 = x^2$
 $x = \pm 3$
 $y = 9 \quad P(3, 9)$

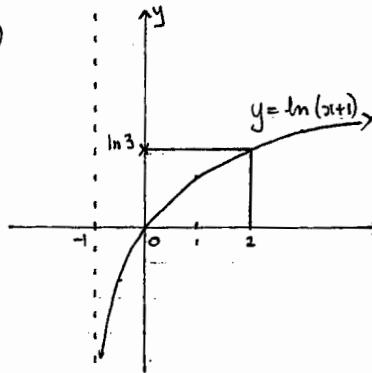
ii) $\int_0^3 x^2 - (2x^2 - 9) dx$
 $\int_0^3 -x^2 + 9 dx = \left[-\frac{x^3}{3} + 9x \right]_0^3$
 $= -9 + 27$
 $= 18 \text{ units}^2$

c) $\sum_{k=4}^{20} 2k-5$

3, 5, 7, ..., 35

$S_n = \frac{n}{2}(a+l)$
 $= \frac{17}{2}(3+35)$
 $= 323$

7d)



$$\text{ii)} P = 283\ 530.74 + 30\ 000 = \$313\ 530.74$$

$$r = 1.0055$$

$$n = 120$$

$$= 313\ 530.74 \times 1.0055^{120}$$

$$= \$605\ 520.87$$

$$\text{iii)} 375 = 2r + l$$

$$l = 375 - 2r$$

$$\pi r \theta = 375 - 2r$$

$$\theta = \frac{375 - 2r}{\pi r}$$

$$\text{diii)} e^y = x + 1$$

$$e^y - 1 = x$$

$$\text{Area} = (2 \times \ln 3) - \int_0^{\ln 3} e^y - 1 \, dy$$

$$= 2 \ln 3 - [e^y - y]_0^{\ln 3}$$

$$= 2 \ln 3 - [(e^{\ln 3} - \ln 3) - (e^0 - 0)]$$

$$= 2 \ln 3 - 3 + \ln 3 + 1$$

$$= 3 \ln 3 - 2$$

/3

Question 10.

$$\text{av)} n = 30 \times 12 = 360^\circ$$

$$r = 6.6\% \div 12 = 0.0055$$

$$a = 250 \times 1.0055$$

$$= \frac{250 \times 1.0055 \times (1.0055^{360} - 1)}{0.0055}$$

$$= \$283\ 530.74$$

$$\text{iv)} \text{Area} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 \left(\frac{375 - 2r}{\pi r} \right)$$

$$= \frac{r(375 - 2r)}{2\pi}$$

$$\text{v)} A = \frac{375r}{2} - r^2$$

$$A' = \frac{375}{2} - 2r$$

$$A'' = -2$$

$$\text{Station points } A' = 0 \quad \frac{375}{2} - 2r = 0$$

$$\frac{375}{2} = 2r$$

$$375 = 4r$$

$$r = 93.75$$

$$\theta = 2$$

$$A = 8789.06$$

$$\text{vi)} 8789.06 = \frac{1}{2} (93.75)^2 \theta$$

$$\theta = 2$$

$$\theta = 114^\circ 35' \text{ or } 115^\circ$$

2 radians is required to produce the maximum area.

$$110^\circ = \frac{11\pi}{8} \quad A = \frac{1}{2} \times (93.75)^2 \times \frac{11\pi}{8}$$

$$= 8436.89$$

110° does not produce maximum area of 8789.0625 m^2